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More Logic Puzzle Apps Solved by Mathematical Programming

Sönke Hartmann

HSBA Hamburg School of Business Administration, Alter Wall 38, D-20457 Hamburg, Germany. ORCID: 0000-0001-6687-1480. E-mail: soenke.hartmann@hsba.de

Abstract. In this paper, we consider six logic puzzles (i.e., single player games) that are available as smartphone apps, namely *Starbattle*, *V3ck*, *Monodot*, *Knight Moves*, *Circuit Scramble* and *Binary Sudoku*. For each puzzle, an integer linear programming formulation is presented (a MathProg implementation is available as well). The purpose is to provide interesting examples and exercises for teaching mathematical programming in OR/MS lectures. The puzzles lead to a broad variety of exercises, ranging from graph models to specific constraint types such as logical and inequality constraints.

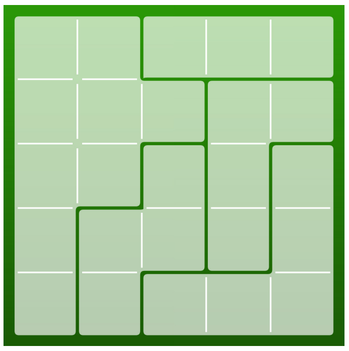
Keywords. Puzzle, teaching modeling.

1 Introduction

Logic puzzles are very popular among computer scientists and operations researchers. Consider, for example, Sudoku which is one of the most liked puzzles. Mathematical programming formulations for Sudoku and related implementations have been published in several scientific journals and books, see Chlond (2005), Koch (2006), Weiss and Rasmussen (2007), Rasmussen and Weiss (2007), Bartlett et al. (2008) and Oki (2012, Section 9.1), and they can be found in various blogs. Further puzzles for which mathematical models have been proposed include, among others, the n queens problem (Letavec and Ruggiero, 2002), other chessboard placement puzzles (Chlond and Toase, 2002; Chlond, 2010), Einstein's riddle (Yeomans, 2003), logic grid puzzles (Chlond, 2014), as well as the popular smartphone puzzle apps flow free (also called number link) and infinity loop (Hartmann, 2018).

One particular motivation for studying logic puzzles in OR/MS is that they can be used when teaching mathematical modeling. It might be interesting for students to develop integer programs to solve puzzles. In recent years, a large number of logic puzzles appeared as smartphone apps, and considering the numbers of downloads reported in the app stores, many of the logic puzzle apps are immensely popular (Hartmann, 2018). It may be particularly motivating for students to develop mathematical models for logic puzzle apps they can easily download and play on their devices.

In this paper, we present six logic puzzles which are available as smartphone apps, and we develop mathematical programming formulations to solve them. As in (Hartmann, 2018), we use the following criteria for selecting the puzzles. Our first goal is to obtain models with different levels of difficulty, with a particular focus on models that are not too difficult (because we want to use them in introductory lectures). The second goal is to include models with a broad range of constraint types. Our third goal is to motivate students to work with the apps. Thus we select puzzles with rules that are simple and easy to learn, and we choose apps that are available for free


(a) Puzzle

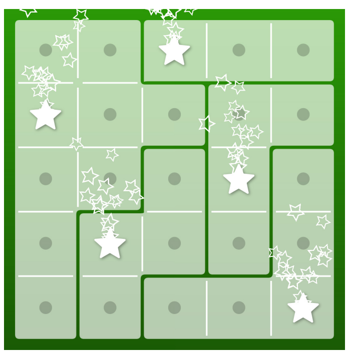

(b) Solution

Figure 1: Smartphone app *Starbattle*

(all apps discussed here are available for Android for free, and some of them are available for iOS as well).

The remainder is organized as follows: In each of the next six sections, a puzzle is introduced, and a mathematical model is developed to solve it. Section 2 presents the app *Starbattle* which leads to a rather easy exercise. Then Sections 3 and 4 discuss the apps *V3ck* and *Monodot* which allow for exercises related to graphs. The next three sections discuss puzzles that are related to specific constraint types. Section 5 deals with puzzles based on the moves of knights on chessboards; this can be used when teaching models for the traveling salesman problem and the related subtour elimination constraints. The app *Circuit Scramble* presented in Section 6 allows to study the formulation of logical constraints (AND, OR, XOR, NOT) in a linear model. In Section 7, a puzzle often called *Binary Sodoku* is considered. One of its rules leads to inequality constraints ($\neq$) which will be covered by the mathematical model. Finally, Section 8 draws some conclusions. Implementations of the models in MathProg (Makhorin, 2010) are available as supplementary material.

2 Starbattle

The app *Starbattle* is based on a grid of $n \times n$ cells. The grid is partitioned into n sets of connected cells; these sets are called islands (see Figure 1). n stars must be placed in the grid such that each row, each column, and each island contains exactly one star. Moreover, all eight cells surrounding a cell with a star must remain empty. In a variant of the puzzle, $2n$ stars are placed, and each row, column and island must contain two stars instead of one.

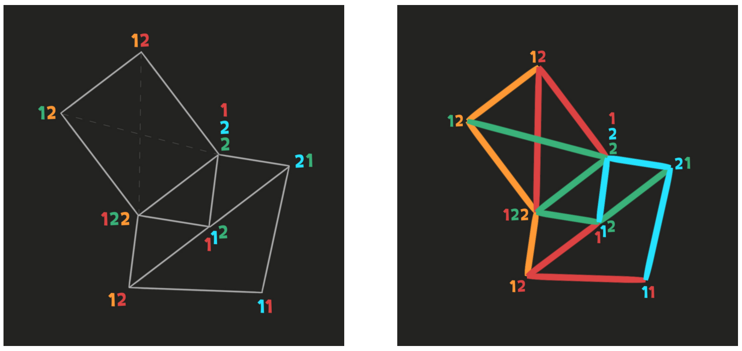
Let S be the number of stars that must be placed in each row, column and island (we have either $S = 1$ or $S = 2$). The set of all cells in the grid is $C = \{(i,j) \mid i,j = 1,\ldots,n\}$. Next, let $A_k \subset C$ be the k-th island, $k = 1,\ldots,n$. We assume that $A_k \cap A_{k'} = \emptyset$ for all $k \neq k'$ and $A_1 \cup A_2 \cup \ldots \cup A_n = C$.

The binary decison variables x_{ij} reflect the placement of the stars:

$$x_{ij} = \begin{cases} 1 & \text{if a star is placed in cell } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

Now we can give the mathematical model for *Starbattle*:

$$\sum_{j=1}^{n} x_{ij} = S \qquad i = 1,\ldots,n \tag{1}$$



(a) Puzzle **(b)** Solution

Figure 2: Smartphone app *V3ck*

$$\sum_{i=1}^{n} x_{ij} = S \qquad j = 1, \dots, n \tag{2}$$

$$\sum_{(i,j) \in A_k} x_{ij} = S \qquad k = 1, \dots, n \tag{3}$$

$$x_{ij} + x_{i+1,j} + x_{i,j+1} + x_{i+1,j+1} \leq 1 \qquad i, j = 1, \dots, n-1 \tag{4}$$

$$x_{ij} \in \{0, 1\} \qquad i, j = 1, \dots, n \tag{5}$$

Constraints (1) and (2) take care of the required number of stars in the rows and columns, respectively. Constraints (3) do the same for the islands. Next, Constraints (4) observe the rule that the cells surrounding a star must be empty. This is achieved by making sure that each 2×2 square of cells contains at most one star. The binary decison variables are declared by Contraints (5). Of course, in case of $S = 1$ Constraints (1), (2) and (5) correspond to those of the assignment problem (Hillier and Lieberman, 2001).

3 V3ck

In the app *V3ck*, a graph (i.e., nodes and edges) is displayed on the screen. A number of different colors is given, and the user has to color each of the edges with one of the colors. Each node is associated with one or more numbers. Each number is displayed in one of the colors and reflects how many of the edges linked to this node have that color. Figure 2 shows a screenshot of the puzzle as well as the related solution. The puzzle is somewhat related to the edge-coloring problem in graph theory (Holyer, 1981). Nevertheless, the constraints of the edge-coloring problem are different (there, adjacent edges must be assigned to different colors, which is not the case for *V3ck*).

For developing a formal model, let n be the number of nodes, and let C be the number of colors. The graph is given by $G = (V, E)$ where $V = \{1, \dots, n\}$ is the set of nodes and E is the set of edges. Parameter a_{ik} determines how many of the edges associated with node i must receive color k.

As can be seen from Figure 2, the puzzle is based on an undirected graph. For modeling purposes, however, it is more convenient to have a directed graph, that is, each edge between nodes i and j is modeled by two directed edges or arcs (i, j) and (j, i). Now the binary decision variables that reflect the assignment of colors to the edges can be defined as follows:

$$x_{ijk} = \begin{cases} 1 & \text{if arc } (i,j) \in E \text{ obtains color } k \\ 0 & \text{otherwise} \end{cases}$$

As before, there is no objective, and we can give the constraints of the mathematical model:

$$\sum_{k=1}^{C} x_{ijk} = 1 \qquad (i,j) \in E \tag{6}$$

$$x_{ijk} = x_{jik} \qquad (i,j) \in E,\ k = 1,\dots,C \tag{7}$$

$$\sum_{(i,j)\in E} x_{ijk} = a_{ik} \qquad i = 1,\dots,n,\ k = 1,\dots,C \tag{8}$$

$$x_{ijk} \in \{0,1\} \qquad (i,j) \in E,\ k = 1,\dots,C \tag{9}$$

Constraints (6) make sure that every arc is assigned exactly one color. Next, Constraints (7) observe that arcs (i,j) and (j,i) must receive the same color (this is only necessary because the model is based on a directed graph). For each node, Constraints (8) check that the number of adjacent edges of each color is correct. Last, the binary decision variables are taken care of by Constraints (9).

4 Monodot

In the app *Monodot*, the player's task is to construct a graph from a set of given nodes such that the specified number of neighbors of each node (i.e., the degree of each node) is met. In the beginning, a grid with $n \times n$ cells is displayed along with a list of nodes. The player has to drag each node to a cell of the grid. When a node is dropped, the app automatically adds an edge to each node already located in a horizontal, vertical and diagonal neighbor cell. Each node contains a number, and when finished, this number must be equal to the degree of the node. Thus, the player has to find a location for each node such that the required number of edges to neighbors is met, see Figure 3.

In addition to the size of the grid n, we need some more notation. The number of nodes requiring t neighbors is denoted as a_t (since a node cannot have more than 8 neighbors in the grid, we have $t = 1,\dots,8$). In the example of Figure 3, we have $a_3 = 4$ and $a_4 = 0$. Let $C = \{(i,j) \mid i,j = 1,\dots,n\}$ be the set of all cells of the grid. We can now define the neighborhood of cell (i,j) by

$$\begin{aligned} N_{ij} = \{&(i-1,j-1),(i-1,j),(i-1,j+1),(i,j-1),\\ &(i,j+1),(i+1,j-1),(i+1,j),(i+1,j+1)\} \cap C \end{aligned}$$

In order to reflect the positioning of nodes in the grid, we introduce the following binary decision variables:

$$x_{ijt} = \begin{cases} 1 & \text{if a node which requires } t \text{ neighbors is placed in cell } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

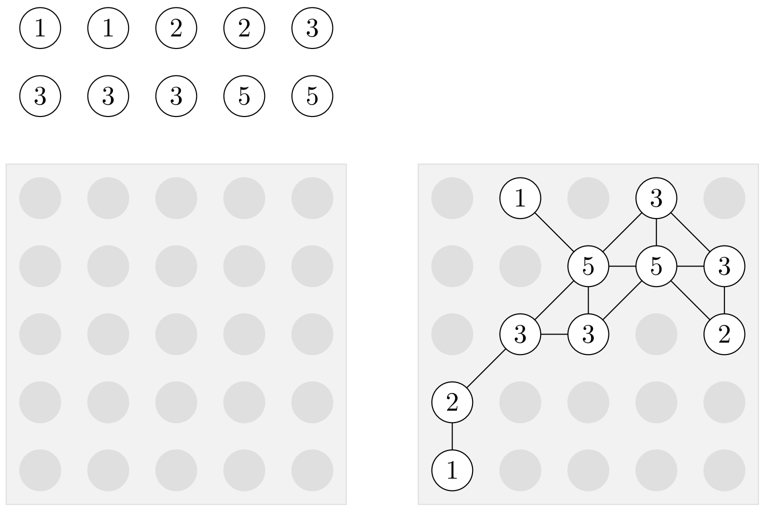
The constraints of the mathematical model are as follows:

$$\sum_{t=1}^{8} x_{ijt} \le 1 \qquad i,j = 1,\dots,n \tag{10}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ijt} = a_t \qquad t = 1,\dots,8 \tag{11}$$

$$\sum_{(k,l)\in N_{ij}} \sum_{t=1}^{8} x_{klt} \ge \sum_{t=1}^{8} t x_{ijt} \qquad i,j = 1,\dots,n \tag{12}$$

$$\sum_{(k,l)\in N_{ij}} \sum_{t=1}^{8} x_{klt} \le \sum_{t=1}^{8} t x_{ijt} + M(1 - \sum_{t=1}^{8} x_{ijt}) \qquad i,j = 1,\dots,n \tag{13}$$



(a) Puzzle **(b)** Solution

Figure 3: Puzzle *Monodot*

$$x_{ijt} \in \{0,1\} \qquad i,j = 1,\dots,n,\ t = 1,\dots,8 \qquad (14)$$

Constraints (10) make sure that each cell of the grid is assigned at most one node. The required number of nodes with t neighbors is taken into account by Constraints (11). Next, Constraints (12) and (13) take care of the desired number of neighbor nodes in the resulting graph. According to (12), a node assigned to a cell has at least t neighbors, but no restriction is imposed if no node is assigned to the cell. Similarly, (13) implies that a node assigned to a cell has at most t neighbors, and again no restriction is made if no node is assigned to the cell (M is a big number and can simply be set to 8). The final Constraints (14) define the binary decision variables.

5 Knight Moves

A well-known puzzle is based on the moves of knights on a chessboard. A knight moves two squares either vertically or horizontally and then one square to the left or to the right. The classical knight-based puzzle is to start in a given square and then successively visit all other squares of the chessboard without visiting any square more than once. There are several variants and extensions, for example rectangular chessboards of different dimensions and irregular chessboards where certain squares may not be visited. Android apps providing knight-based puzzles include *Knight's Alley*, *Knight Hopper*, *Knight's Move*, and *Knight Ride*. Several algorithms for constructing a knight's tour (that is, a knight's Hamiltonian path) on chessboards have been proposed in the literature, see, e.g., Conrad et al. (1994) and Lin and Wei (2005).

In what follows, we consider a rectangular board with m rows and n columns (of course, the classical chessboard with $m = n = 8$ is included as a special case). The knight is placed on a square (i_s, j_s) from which it has to start. All squares must be visited exactly once in a continuous sequence of knight moves.

Let us define a graph $G = (V, E)$ where each node $(i,j) \in V$ corresponds to a square on the chessboard. For each node (i,j) the set of neighbor nodes is given by

$$\begin{aligned} N_{ij} = \{&(i-2,j-1),(i-2,j+1),(i-1,j-2),(i-1,j+2),\\ &(i+1,j-2),(i+1,j+2),(i+2,j-1),(i+2,j+1)\} \cap V \end{aligned}$$

Of course, the neighbor nodes of node (i,j) represent those squares that can be reached from square (i,j) by a knight in a single move. The set E of edges is now defined by introducing an edge (i,j,k,l) between any two neighboring nodes $(i,j) \in V$ and $(k,l) \in N_{ij}$. This way, the edges represent the possible knight moves.

We need two types of decision variables. Variables x_{ijkl} with $(i,j) \in V$ and $(k,l) \in N_{ij}$ are binary and reflect the edges corresponding to the selected moves of the knight:

$$x_{ijkl} = \begin{cases} 1 & \text{if the knight should move from square } (i,j) \text{ to square } (k,l) \\ 0 & \text{otherwise} \end{cases}$$

Moreover, we introduce variables $z_{ij} \geq 0$ for all nodes $(i,j) \in V$. They will be needed for a constraint which ensures that a single connected sequence of moves is generated. Now the contraints can be given as follows (again, an objective function is not needed):

$$\sum_{(k,l)\in N_{i_s,j_s}} x_{k,l,i_s,j_s} = 0 \tag{15}$$

$$\sum_{(k,l)\in N_{i_s,j_s}} x_{i_s,j_s,k,l} = 1 \tag{16}$$

$$\sum_{(k,l)\in N_{ij}} x_{klij} = 1 \qquad (i,j) \in V \setminus \{(i_s,j_s)\} \tag{17}$$

$$\sum_{(k,l)\in N_{ij}} x_{ijkl} \leq 1 \qquad (i,j) \in V \setminus \{(i_s,j_s)\} \tag{18}$$

$$z_{kl} - z_{ij} + M(1 - x_{ijkl}) \geq 1 \qquad (i,j) \in V,\ (k,l) \in N_{ij} \tag{19}$$

$$x_{ijkl} \in \{0,1\} \qquad (i,j) \in V,\ (k,l) \in N_{ij} \tag{20}$$

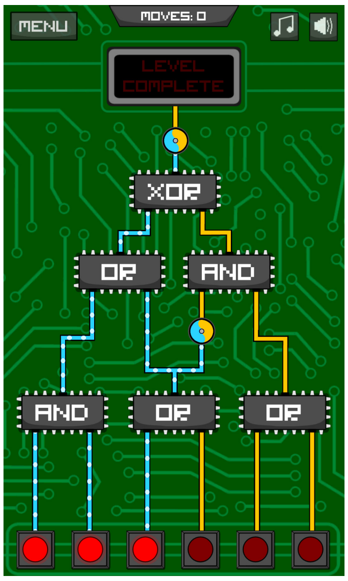
$$z_{ij} \geq 0 \qquad (i,j) \in V \tag{21}$$

Constraints (15) and (16) make sure that there is no move to the start square and that there is one move from the start square, respectively. The next two types of contraints take care of the flow through the remaining squares. Constraints (17) observe that each such square will be visited. Moreover, Constraints (18) control that a square can only be left at most once (note that the last square cannot be left). In addition, we need Constraints (19) which mean that each node (i,j) is assigned a z value that is larger than that of its predecessor node (thereby, M is a large number which can be set to $m \cdot n$). This way, cycles are made impossible, and a single connected sequence of moves is obtained. Note that these are the subtour elimination constraints suggested by Miller et al. (1960) for the traveling salesman problem (TSP). Finally, the decision variables are declared by (20) and (21).

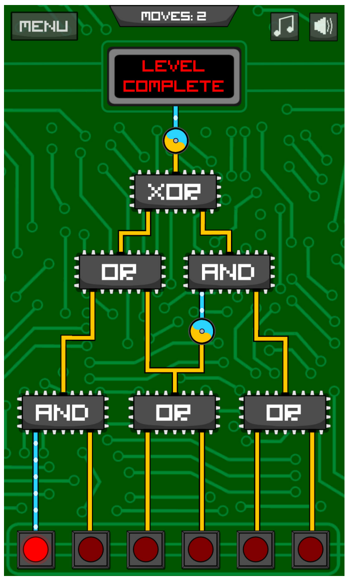
It may be mentioned that extensions of the knight move problem as given above can be incorporated into the model as well. In particular, irregular chessboards where certain squares may not be visited can be modeled by adapting graph G. Denoting the set of nodes reflecting the forbidden squares as F, we simply define V to contain all nodes except those included in F. The definition of the set of edges E given above needs not be modified. The model (15)–(21) automatically takes forbidden squares into account if V is adapted accordingly.

6 Circuit Scramble

The app *Circuit Scramble* displays a circuit which consists of logic gates, see the screenshots in Figure 4. There are AND, OR and XOR gates, each with two inputs, and NOT gates with one input (the latter are called inverters in the app and are depicted as circles). Moreover, the circuit contains several inputs as well as one output node. The player has to decide which inputs have to be switched on (set to TRUE) or off (FALSE) such that the output node is TRUE and shows "level complete." The goal is to achieve this by changing the states of the smallest possible number of inputs.



(a) Puzzle



(b) Solution

Figure 4: Smartphone app *Circuit Scramble*

We consider a graph with n nodes labeled $i = 1, \ldots, n$. Each input of the circuit as well as each output of a gate is reflected by a node. The set of nodes is now partitioned into subsets. I^1 is the set of the input nodes which are TRUE in the initial state, and I^0 is the set of the input nodes which are FALSE in the initial state. The remaining sets reflect the outputs of the different gate types. A contains the AND gates, O contains the OR gates, X contains the XOR gates, and N contains the NOT gates. Next, the circuit structure is considered. Let p_i and q_i be the two predecessor nodes (or inputs) of each AND, OR, and XOR gates, $i \in A \cup O \cup X$, and let p_i be the single predecessor (or input) of each NOT gate, $i \in N$. We assume that node n is the output node of the circuit which must be switched on.

Considering the screenshot of Figure 4, we assume that we label the nodes from bottom left to top right. Thus, the six inputs would be indexed by $i = 1, \ldots, 6$, and the AND gate at the bottom on the left would be $i = 7$. Then we have $7 \in A$, and the predecessors of this gate are $p_7 = 1$ and $q_7 = 2$.

We introduce straightforward binary decision variables to reflect the logical state of each node $i = 1, \ldots, n$:

$$x_i = \begin{cases} 1 & \text{if node } i \text{ should be TRUE} \\ 0 & \text{otherwise} \end{cases}$$

Now we are ready to define the model which, unlike the previous models, contains an objective function:

$$\text{Minimize} \sum_{i \in I^1} (1 - x_i) + \sum_{i \in I^0} x_i \tag{22}$$

subject to

$$x_i \leq x_{p_i} \quad x_i \leq x_{q_i} \quad x_i \geq x_{p_i} + x_{q_i} - 1 \qquad i \in A \tag{23}$$

$$x_i \geq x_{p_i} \quad x_i \geq x_{q_i} \quad x_i \leq x_{p_i} + x_{q_i} \qquad i \in O \tag{24}$$

$$x_i \leq x_{p_i} + x_{q_i} \quad x_i \geq x_{p_i} - x_{q_i} \quad x_i \geq x_{q_i} - x_{p_i} \quad x_i \leq 2 - x_{p_i} - x_{q_i} \qquad i \in X \tag{25}$$

$$x_i = 1 - x_{p_i} \qquad i \in N \tag{26}$$

$$x_n = 1 \tag{27}$$

	0						1
0		0		1			
					1	1	
0	0			1			
0		1				1	0
			1			0	
				1		0	
1			1				0

(a) Puzzle

1	0	1	0	0	1	0	1
0	1	0	1	1	0	1	0
1	0	0	1	0	1	1	0
0	0	1	0	1	1	0	1
0	1	1	0	1	0	1	0
1	0	0	1	0	1	0	1
0	1	1	0	1	0	0	1
1	1	0	1	0	0	1	0

(b) Solution

Figure 5: Binary Sudoku

$$x_i \in \{0,1\} \qquad i = 1,\ldots,n \tag{28}$$

Objective (22) minimizes the deviation from the initial state of the inputs and thus the number of moves. Constraints (23), (24), (25) and (26) take care of the AND, OR, XOR, and NOR gates, respectively. Next, Constraint (27) makes sure that the output of the circuit is TRUE, that is, the sign "level complete" is illuminated. The binary decision variables are defined by (28).

7 Binary Sudoku

A puzzle often referred to as *Binary Sudoku* includes a grid of $n \times n$ cells, some of which contain a 0 or a 1 (n is an even number). The task is to fill the grid by entering either a 0 or a 1 into each empty cell. The following rules have to be observed: First, each row and each column must contain exactly the same number of 0s and 1s. Second, no more than two consecutive cells (either horizontally or vertically) may contain the same number. Third, no two rows may be exactly the same, and no two columns may be identical. Figure 5 shows an example puzzle along with the solution.

There are many apps for this puzzle. Apps for Android include *Mr. Binairo*, *Binaris 1001*, *LogiBrain Binary*, *Zuzu Binary Sudoku Puzzle*, *Binoxxo* and several others. Some apps use other symbols than 0 and 1 or two different colors, but the logic remains the same.

In addition to the size of the grid n, we need some notation to specify the initial state of the grid. Let A be the set of cells (i,j) that contain a 0 at the beginning, and let B be the set of cells that contain a 1.

We introduce binary decision variables x_{ij} that determine whether a 0 or a 1 should be entered into cell (i,j). Moreover, we need binary variables y_{ik} for constraints that ensure that rows $i = 1,\ldots,n$ and $k = i+1,\ldots,n$ are different. Likewise, binary variables z_{ik} are required for constraints that verify that columns $j = 1,\ldots,n$ and $k = j+1,\ldots,n$ are different. Now the constraints can be formulated as follows:

$$x_{ij} = 0 \qquad (i,j) \in A \tag{29}$$

$$x_{ij} = 1 \qquad (i,j) \in B \tag{30}$$

$$\sum_{j=1}^{n} x_{ij} = \frac{n}{2} \qquad i = 1,\ldots,n \tag{31}$$

$$\sum_{i=1}^{n} x_{ij} = \frac{n}{2} \qquad j = 1,\ldots,n \tag{32}$$

$$1 \leq \sum_{k=j}^{j+2} x_{ik} \leq 2 \qquad i = 1, \dots, n,\ j = 1, \dots, n-2 \tag{33}$$

$$1 \leq \sum_{k=i}^{i+2} x_{kj} \leq 2 \qquad i = 1, \dots, n-2,\ j = 1, \dots, n \tag{34}$$

$$\sum_{j=1}^{n} 2^{j-1} x_{ij} - \sum_{j=1}^{n} 2^{j-1} x_{kj} \leq M y_{ik} - 1 \qquad i = 1, \dots, n,\ k = i+1, \dots, n \tag{35}$$

$$\sum_{j=1}^{n} 2^{j-1} x_{ij} - \sum_{j=1}^{n} 2^{j-1} x_{kj} \geq 1 - M(1 - y_{ik}) \qquad i = 1, \dots, n,\ k = i+1, \dots, n \tag{36}$$

$$\sum_{i=1}^{n} 2^{i-1} x_{ij} - \sum_{i=1}^{n} 2^{i-1} x_{ik} \leq M z_{jk} - 1 \qquad j = 1, \dots, n,\ k = j+1, \dots, n \tag{37}$$

$$\sum_{i=1}^{n} 2^{i-1} x_{ij} - \sum_{i=1}^{n} 2^{i-1} x_{ik} \geq 1 - M(1 - z_{jk}) \qquad j = 1, \dots, n,\ k = j+1, \dots, n \tag{38}$$

Constraints (29) and (30) take the zeros and ones of the initial state of the grid into account. Next, Constraints (31) and (32) make sure that there are as many zeros as ones in each row and each column. The rule that no more than two adjacent cells may contain the same number is taken care of by Constraints (33) for rows and by (34) for columns, respectively. They control that the sum of any three neighboring cells must be at least 1 (which avoids three zeros) and at most 2 (which avoids three ones). The requirement that all rows must be different is considered by Constraints (35) and (36), while Constraints (37) and (38) do the same for columns. The binary vectors of the rows and columns are transformed into integers, and the constraints then make sure that these integers are not equal. With the help of additional variables y_{ik} and z_{ik} we obtain constraints reflecting inequality ($\neq$). Thereby, M is a big number and can be set to $M = 2^{n+1}$. It should be noted that these constraints to achieve inequality only work for integer values.

8 Conclusions

We have presented six logic puzzles which are available as smartphone apps, along with mathematical models to solve them. As in Hartmann (2018), the puzzle apps and models are used in the lecture *Operations Research* in two BSc programs, namely business administration and business informatics. The lecture is given in the fourth semester, after the students have completed basic lectures on mathematics.

The main purpose when using puzzles is to provide interesting examples for in-class discussions of modeling approaches as well as further exercises that students can do at home. In order to encourage students who are less familiar with mathematical formalisms, we often start with the development of models for given instances (rather than general models). As instances, we simply use the screenshots given in this paper. Working with instances is perceived as being much easier than working with general models by most students.

Moreover, it should be mentioned that discussing logic puzzles can help to demonstrate the broad applicability of integer linear programming and the power of the algorithms included in solvers. In our lectures, we emphasize that variables, linear constraints and objective can in fact be seen as a specific "language," and that once a problem has been translated to that language, standard algorithms can be used to solve it. We point out that while mathematical programming is fairly restricted because it only allows for linear expressions, it is also very general because it can solve many different types of problem settings. Logic puzzles are used to emphasize that this is not limited to typical OR problems.

Of course, the main goal when teaching mathematical programming (and OR in general) is to enable students to identify problems in practice and to solve them. Beyond that, however, another goal is to improve the general analytical skills of the students. In this context, we find that modeling logic puzzles is a valuable addition to the modeling of the usual OR problems.

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