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Investing in Times of Inflation Fears: Diversification Properties of Investments in Liquid Real Assets

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Abstract

The financial crisis and the rescue measures taken by governments and central banks increased investors' interest in liquidity and in real assets supposed to offer a hedge against inflation. Against this background, we investigate empirically four real assets (real estate, commodities, infrastructure, and shipping) for which there are investment instruments available which trade in liquid markets.

Our empirical study using data from 1999 to 2009 yields several results: First, in most cases, the addition of real assets improved portfolio performance (measured with Sharpe ratio, Sortino ratio, Omega ratio, and Modified Sharpe ratio) in comparison with a base portfolio consisting only of standard stocks and bonds. Among the four real assets, infrastructure and shipping clearly outperformed commodities and real estate. Second, the time frame chosen for the analysis matters very much. This is bad news for investors because there is no such thing as the single 'true' time frame for this purpose. Due to our analytical approach, we regard our conclusions, in spite of their general time dependence, as rather solid. Third, despite great conceptual differences, our four performance measures lead to the same conclusions. This result is interesting for investors beyond our specific setting because the selection of a specific performance measure from the vast supply of such measures does not seem to matter much.

JEL Classification: G11, G15

Keywords: diversification, inflation, portfolio choice, investments in real assets

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INTRODUCTION

Investment decisions are always influenced by a variety of factors. Currently, the fear of inflation is a major consideration of investors in light of the extremely expansive monetary and fiscal policies to fight the global economic crisis. In addition, as the crisis has raised uncertainty, the importance of liquidity has grown as well because investors want to be able to react to changing circumstances quickly. A third factor to be mentioned is diversification. The crisis demonstrated impressively both the benefits and the limits of almost all assets' diversification properties.

This article takes the position of investors whose considerations are mainly driven by the three factors mentioned. To protect their wealth from inflation, investors are given advice to shift more of their funds to real assets. Of course, stocks are the premier and classic means to acquire a liquid participation in real assets. But despite the stock markets' impressive rebound in 2009, investors' confidence in stock investments is not fully restored. Therefore, other real assets are often mentioned as protection from inflation and thus are attractive to investors. Moreover, for some of these assets, markets and investment instruments are available whose liquidity does not lag very much behind average stock markets. What is more, investments in assets other than standard bonds and standard stocks were promoted already before the crisis. It was said that adding such assets — some of them labeled as alternative assets — to a standard portfolio would improve diversification (see, e.g., for hedge funds Amin & Kat [2002; 2003], French [2005], Heidorn et al. [2007]; for private equity see, for instance, Wahrenburg et al. [2003], Emery [2003], Milner & Vos [2003]; for real estate mixed results could be shown by, e.g., Maurer & Sebastian [1999], Brounen & Eichholtz [2003], Chan et al. [2003], Hübner et al. [2004]; for commodities see, for instance, Anson [2006]; regarding shipping investments, see Kavussanos et al. [2003], Gong et al. [2006], Grelck et al. [2009], Drobetz et al. [2010]; considering infrastructure investments, see Inderst [2009]).

Our scenario is prominently shaped by inflation fears, the desire for liquidity, and the wish for diversification. Against this scenario, this article assesses the investment advice given by many consultants, i.e., to invest a share of one's wealth into liquid real assets other than stocks [e.g., see Amenc et al. [2009], O´Donnell III [2009], Bräuninger et al. [2009]). The analysis comprises three liquid real assets often mentioned in such recommendations, i.e., commodities, real estate, and infrastructure; in addition, we also analyze investments in shipping. For real estate and commodities, there are numerous empirical studies in that issue available and the majority support the hypothesis that these assets indeed provide some inflation hedge (on real estate, see, for instance, Liu et al. [1997], Hamelink et al. [1997], Mull & Soennen [1997], Stevenson [2000], Hudson-Wilson et al. [2003], and Amenc et al. [2009]; on commodities, see, e.g., Anson [2006: 333f.], Gorton & Rouwenhorst [2006], and Amenc et al. [2009]). Empirical investigations of the link between inflation and shipping returns are rare; so far, they did not ascertain such connection (see, for instance, Grammenos & Arkoulis [2002] and Drobetz et al. [2010]). The stock of empirical evidence for infrastructure is even smaller. Here we can only refer to bank publications underlining that infrastructure cash flows are often linked to inflation, thus providing some inflation hedge (see, e.g., Mansour & Nadji [2007] from Deutsche Bank Group and Eagar [2008] from Magellan Asset Management].

Two questions will be dealt with empirically in greater detail: 1) No investor would hold a portfolio which only contains the real assets mentioned above. Instead, these assets should be regarded as an addition to a base portfolio which, even in current times, builds on standard bonds and standard stocks. Therefore, we explore how the addition of liquid real assets affects the risk-return profile of the base portfolio. 2) Analysis of the diversification properties inevitably requires the usage of data from the past. Moreover, academic and applied research developed a great variety of measures to analyze diversification properties. For this reason, we apply the most popular measures to numerous periods and sub-periods. By doing so, we can assess whether the answer to the first question crucially depends on the measure applied and the time period explored or whether the recommendation derived from such calculations is robust and reliable. The results for our second question are of general interest that goes beyond our specific investment setting.

The analysis starts with the discussion of the risk-return measures (Sharpe ratio, Modified Sharpe ratio, Sortino ratio, and Omega ratio) and the formulation of the relevant hypotheses. Then we describe the database. After that, the diversification properties of investments in commodities, real estate, infrastructure, and shipping will be analyzed. To test our hypotheses, we compare two portfolios: The *base portfolio* consists of standard bonds and standard stocks. The composition of the *enhanced portfolios* contains an investment in a single real asset as a third component. The third component is a liquid investment instrument representing either real estate, commodities, infrastructure or shipping.

PERFORMANCE MEASURES

The performance of an asset class is a function of mean and risk of returns. The assumption about the return distribution plays a large role for the usability of different performance measures. It is often assumed that returns are normally distributed. In this case, the distribution can be fully described by its first two moments: mean and variance. Unfortunately, historically financial markets have shown much higher volatility than estimated, e.g., 1987, 1989, 2008. For that reason, higher moments of the return distribution have to be considered if the return distribution is not normal. These measures would be skewness and kurtosis.

Skewness is the third standardized moment (for details, see DeFusco et al. [2004: 144-149]). It measures the asymmetry in the probability distribution. For a normally distributed random variable skewness is equal to zero. Otherwise, if the skewness is different from zero, it implies that observations are not spread symmetrically around the mean. If the skewness is larger than zero the right tail of the distribution is longer. This is characteristic for a lot of small losses but larger gains. If the skewness is lower than zero the left tail of the distribution is longer. This is typical for a lot of small gains but larger losses.

Kurtosis is the fourth standardized moment. The kurtosis of a normally distributed variable is equal to three. To correct this, usually instead of kurtosis the excess kurtosis is calculated, which for a normally distributed variable has the value of zero (for details, see DeFusco et al. [2004: 149-153]).

Kurtosis measures the peakedness or heaviness of a distribution. If excess kurtosis is larger than zero, the distribution is called leptokurtic. The shape of a leptokurtic distribution has a more acute peak and fat tails. In terms of probability it implies a higher chance of values around the mean, but at the same time also a higher chance of extreme values. If excess kurtosis is lower than zero, the distribution is called platykurtic. Platykurtic distribution exhibits a flat shape with a smaller peak and thin or no tails. It involves a lower probability of values around the mean and of extreme values and, in return, a higher probability of moderately deviated values from the mean. The kurtosis risk is also known as "fat tail risk".

This article covers four performance indicators: Sharpe ratio, assuming normal distribution of return series, Sortino ratio, taking skewness risk into account, as well as Omega ratio and Modified Sharpe ratio, which reflect in their measurement both skewness and kurtosis risks.

Sharpe Ratio

One of the most common measure is the Sharpe ratio (Sharpe [1966]). It is an effective tool for investment evaluation due to the fact that it expresses the mean and the risk characteristics of asset or portfolio returns in a single number. It relates the excess return of an investment position to its risk. For realized returns, the Sharpe ratio of a portfolio P can be written as:

$$S_p = \frac{r_P - r_f}{\sigma_P} \quad \textbf{(1)}$$

where:

S_P Sharpe ratio of portfolio P

r_P mean return of portfolio P

σ_P standard deviation of portfolio P

r_f risk-free rate of return

The underlying logic of the Sharpe ratio is easily comprehensible and its computation is simple, the fact which assured its popularity so far. For all these advantages, the assumption of normal return distribution represents the very Achilles heel of the Sharpe ratio, triggering both criticism and the search for alternative performance indicators.

Sortino Ratio

The Sortino ratio defines the term risk in a different way. Instead of looking at the fluctuation around the mean, only negative deviations from a minimum required return are relevant (downside deviation). Therefore, the Sortino ratio is defined as the excess return over a minimum acceptable return divided by the downside deviation (Sortino/Price [1994]).

$$SortinoRat\ io = \frac{r_p - MAR}{DD_p} \quad \textbf{(2)}$$

where:

r_P mean return of portfolio P

MAR minimum acceptable return

DD downside deviation

The downside deviation can be calculated as

$$DD = \sqrt{\frac{\sum_{\substack{i=1 \\ r_i < MAR}}^{n} \left(r_i - MAR\right)^2}{n}} \qquad \textbf{(3)}$$

where:

n number of observations

r_i rate of return in the time period i

Only those returns have to be considered which fall below a certain target level designated as minimum acceptable return. One deficit of this ratio is related to the estimation of the MAR. Since different people might have different expectations about this return, it influences directly the downside deviation and therefore leads to difficulties in performance comparison. For this reason, we follow the common procedure to utilize the risk-free rate of return as MAR in our empirical analysis. While Sortino ratio addresses the limitations of classical performance measurement regarding the skewness risk, it still leaves the problem of integrating the kurtosis risk unsolved. The following two performance indicators attempt to cope with both of these risks.

Omega Ratio

The Omega ratio considers both skewness and kurtosis. It can be calculated in the following way (De Souza/Gokcan [2004]):

$$\Omega(t) = \frac{\sum_{i=1}^{n} \max\left(0; r_i - T\right)}{\sum_{i=1}^{n} \max\left(0; T - r_i\right)} \qquad \textbf{(4)}$$

where:

n number of observations

r_i rate of return in the time period i

T threshold return

The ratio shows the excess return above a minimum return in relationship to the loss with the same threshold. The optimal portfolio has the highest Omega ratio. The higher it is, the more density is concentrated to the right side of the threshold. The Omega ratio decreases naturally with an increasing threshold. Nevertheless, the ranking of investments based on the Omega ratio may vary with different threshold values. Bacmann and Scholz [2003] argue that for the purpose of investment evaluation the Omega measure should be calculated for thresholds between 0% and the risk-free rate. In our empirical analysis, the threshold return is defined as the average of 0% and the risk-free rate.

Modified Sharpe Ratio

The underlying logic of the Modified Sharpe ratio is similar to that of classical Sharpe ratio, namely it describes the excess return in relation to risk. However, risk is not measured by standard deviation anymore, but by Modified Value-at-Risk, which in turn is derived from the classical Value-at-Risk (VaR).

VaR represents the loss, which is not exceeded with a given probability of $1 - \alpha$, which corresponds to the confidence level (Blum et al. [2003]). VaR is formulated as follows:

$$VaR = -\left(\mu + z_\alpha \times \sigma\right) \quad \textbf{(5)}$$

where:

μ average return

z_α the α quantile of the standard normal distribution

σ standard deviation

Since the classical VaR assumes also a normal distribution of returns, a modified version has to be used (Favre/Galeano [2002]). Similarly to VaR, its modified version determines the worst usually 5% or 1% returns, according to the confidence level, however, it additionally integrates in its output the skewness and kurtosis of the return distribution. In the empirical analysis, we set the confidence level at 5%.

$$MVaR = -\left(\mu + \left(z_\alpha + \frac{(z_\alpha^2 - 1)\times\lambda_1}{6} + \frac{(z_\alpha^3 - 3z_\alpha)\times\lambda_2}{24} - \frac{(2z_\alpha^3 - 5z_\alpha)\times\lambda_1^2}{36}\right)\times\sigma\right) \quad \textbf{(6)}$$

where:

λ_1 skewness

λ_2 excess kurtosis

The Modified Sharpe ratio can be calculated by substituting the standard deviation from the denominator with the Modified VaR (MVaR). Thus, the Modified Sharpe ratio equals to (Gregoriou/Gueyie [2003]):

$$Modified\ \ Sharpe\ \ ratio = \frac{r_p - r_f}{MVaR} \quad \textbf{(7)}$$

where:

r^P mean return of portfolio P

r^f risk-free rate of return

MVaR modified value-at-risk

In the case of normally distributed return series, the Modified Sharpe ratio converges to the classical Sharpe ratio. The benefits of the modified version are revealed in the case of non normally distributed returns, since it encompasses the four moments of distribution (mean, variance, skewness, and kurtosis) when evaluating the performance of an investment.

DATA BASE

In order to assess whether investments in liquid real assets might enhance the diversification properties of a traditional stock and bond portfolio, it is necessary to define the traditional asset classes already available to investors. In our investment universe, investors may invest in a well diversified stock portfolio and in a well diversified bond portfolio. The stock portfolio is represented by the MSCI World Index. The index is designed to measure global developed market equity performance. The J.P. Morgan Global Bond Index serves the same task for bonds (J.P. Morgan [2009]). Data are from the website of MSCI Barra (www.mscibarra.com, visited at 13.1.2010) and Datastream, respectively.

Next, the indexes representing our four real assets will be explained. Note that all are based on liquid investment instruments. The GPR 250 Property Securities Index consists of the 250 most liquid property companies worldwide (Global Property Research [2009]). Companies are included for which at least 75% of operational turnover is derived from investment activities (property investment companies) or investment and development activities combined (hybrid property companies).

The NMX30 Infrastructure Global consists of the 30 largest and most liquid basic infrastructure companies (LPX Group [2009]). The index is diversified across countries, currencies, and infrastructure industries. The following infrastructure sectors are covered: toll roads/bridges, airports, ports, pipeline networks (water, gas, oil) and communication networks. Eligible companies for inclusion in the NMX base universe show a minimum basic infrastructure (network) revenue contribution of at least 50%.

The Thomson Reuters/Jefferies CRB Index (TR/J CRB) is a commodity price index (Jefferies & Company [2009]). Founded in 1957, the Reuters CRB Index has a long history as the most widely followed index of commodities futures. The Reuters/Jefferies CRB Index uses a four-tiered approach to allocating among the commodities included in the Index. Group I includes only petroleum products with a weight of 33%; Group II includes seven commodities which are highly liquid with a weight of 42%; Group III is comprised of four liquid commodities with a weight of 20%; Group IV includes five commodities that may provide valuable diversification with a weight of 5%. All commodities are equally weighted within Groups II, III and IV.

Our research shipping index is based on the stocks of container, tanker, and bulker companies that make up the following indexes and stock lists, respectively: Clarkson Liner Share Price Index and Clarkson Tanker Share Price Index (www.clarksons.com, visited at 21.12.2009); dry bulk stocks listed in Dry Bulk Insight, a monthly report published by Drewry Publications (www.drewry.co.uk, visited at 21.12.2009). Additionally, the shipping news service Trade Winds was considered (www.tradewinds.no, visited at 21.12.2009). We select a group of 44 international stocks for which return data were available for the full sample period from January 1999 to December 2009 and calculate an equally-weighted index. For 1999 to 2001, monthly data are available, after that we dispose of daily return data.

All indexes enter the analysis as performance indexes and on a US$ basis. Data are from Datastream. The risk-free rate is approximated by 1-month-LIBOR and by 1-day-LIBOR, respectively, also on a US$ basis. Interest rate data come from Economagic. Due to some data availability limitations with respect to our shipping index, all analyses including the years 1999 to 2001 are based on monthly return data, time frames starting not before 2002 use daily return data. We analyze discrete returns since they are more robust when used for calculations in a portfolio context than continuous returns (Poddig et al. [2000: 151-152]).

HYPOTHESES

Already before the crisis, asset managers' and investors' attention to real assets has been increasing. The main reason for this being the potential diversification benefit ascribed to these assets. If their correlation to traditional assets such as standard stocks and standard bonds were sufficiently low, more efficient mean-risk combinations of returns could be achieved by investments in these asset classes. Hence, our first hypothesis is formulated as follows:

Hypothesis 1: Adding a real-asset component to traditional stock and bond portfolios enables investors to achieve more efficient mean-risk combinations of returns.

Evaluating the diversification properties of any asset inevitably requires to determine two points: the time span of past data to be used and the performance measure to be applied. As there are numerous combinations of time span and performance measure available, it is important for investors to know whether the result, i.e., the assessment of a specific asset's diversification properties, crucially depends on the specific features of the analysis or whether it is rather stable. In the latter case, it seems more useful for future investment decisions. Therefore, our second and third hypothesis read as follows:

Hypothesis 2: The empirical assessment of an asset's diversification properties is quite robust with respect to the performance measure applied (performance-measure sensitivity).

Hypothesis 3: The empirical assessment of an asset's diversification properties is quite robust with respect to the time span of past data used (time sensitivity).

DESCRIPTIVE STATISTICS

Exhibit 1 provides descriptive statistics for the monthly return data of the full sample period, namely mean, standard deviation, skewness, and excess kurtosis, as well as the results of the Jarque-Bera test analyzing whether return data are normally distributed. The numbers reveal that shipping earned the highest monthly return (2.63%) and the MSCI World the lowest (0.3%). As a consequence of the financial crisis, the latter is lower than the average return of the bond index. Turning to the standard deviation, the maximum can be found for shipping and the minimum for the bond index. A rather favorable relation of mean and standard deviation can be measured for infrastructure (mean 0.93%, standard deviation 4.4%).

Exhibit 1 about here

The standard deviation is an appropriate measure of risk in case of normally distributed returns. In Exhibit 1, only the bond index returns feature a normal distribution. For all other assets, the hypothesis of normally distributed returns can be rejected according to the Jarque-Bera test at a significance level of 1%. This characteristic is reflected in skewness and kurtosis. Accordingly, with the exception of the bond index, the indexes are negatively skewed. Compared to a normal distribution, probability of negative returns and thus of losses is higher. Moreover, for all indexes positive excess kurtosis can be measured. However, for bonds it has by far the lowest value. Positive excess kurtosis indicates higher probabilities for larger gains and losses in comparison with a distribution without excess kurtosis.

In the following analyses, we do not only look at the whole sample period, but also at a great number of sub-periods to evaluate the time stability of results. We explore each of the years from 2002 to 2009 separately, we look at a long bear market (31.3.2000-31.3.2003) and a long bull market (31.3.2003-31.10.2007), and last but not least we investigate the current crisis in a longer time frame (30.11.2007-31.12.2009) and in a shorter one (31.5.2008-31.12.2009). For the full sample period, the bear market period, and the bull market period, the analyses are based on monthly return data; for all other sample periods, daily return data are available.

Exhibit 2 provides a space-saving survey of the normality tests of returns for all sample periods. It reveals that the results for the complete sample period are not unusual since not normally distributed returns are far more prevalent. Another interesting feature is that with only a single exception all returns are normally distributed during the bear and the bull market period. This is in stark contrast to all other time frames, in particular to those covering the financial crisis. From an analytical point of view it is a favorable feature of the data that there are sample periods with normally distributed returns as well as periods in which returns are not normally distributed. Thus the data base enables us to explore whether the results for performance measures are truly sensitive to return distributions as assumed in theory.

Exhibit 2 about here

ANALYSIS OF DIVERSIFICATION PROPERTIES

Approach

The starting point of our analysis is an investor who invests 50% of his funds in a diversified international stock portfolio, represented by the MSCI World Index, and 50% in a diversified international bond portfolio, which is represented by the J.P. Morgan Global Bond Index. This portfolio is the base portfolio.

Moreover, there are many enhanced portfolios. The real-world investor guided our considerations concerning the enhanced portfolio: Realistically, participation in a single class of real assets cannot be more than a supplement of a rather limited size to a widely diversified portfolio like the base portfolio. For instance, applying an efficiency-based portfolio composition rule to past data might suggest to invest 80% in infrastructure and 20% in bonds. Such a portfolio composition would not be realistic. Instead, it is to be expected that investors place their funds only to a limited extent in real assets, using the base portfolio as a starting point to make cautious alterations. Hence, we limit the portfolio weight of real assets to 20%. In addition, we assume that investors do not want to increase their stake in riskier assets. As a consequence, we increase the weight of the real asset in steps of five percentage points to 20%, simultaneously reducing the share of the MSCI World Index accordingly. Thus, the relative proportion between riskier investments in stocks and real assets on the one hand and bond investment on the other hand remains unchanged. Finally, to keep things simple we only allow for one real asset in the portfolio at the same time, i.e., our enhanced portfolio is always a three-component portfolio consisting of 50% in bonds, and 50% in stocks and a single real asset with a 20% maximum share in the real asset.

In total, we analyze 17 portfolios (base portfolio and four enhanced portfolios for each of the four real assets which have a stake of 5%, 10%, 15%, and 20%) with four performance measures (Sharpe ratio, Modified Sharpe ratio, Sortino ratio, and Omega ratio). To investigate the time sensitivity of our result, we conduct this analysis not only for our complete sample period from 1999 to 2009. In addition to that, we divide our complete sample period into 12 sub-periods, as explained in the descriptive statistics. Thus, we can compare the diversification properties of our 17 portfolios as reflected in four performance measures for 13 time frames (complete sample period and 12 sub-periods). The availability of results for 13 time frames enables us not only to assess time sensitivity of our results. Since the descriptive statistics revealed that the return data are normally distributed for some time frames but not for all of them, and since the four performance measures differ in terms of the assumed return distribution, our data configuration also allows to evaluate the distribution sensitivity of the results.

Results

Exhibit 3 exemplifies for the full sample period the kind of analysis we conduct for all 13 time frames. The exhibit depicts for all 17 portfolios the absolute value of each of the four performance measures as well as the ranking of the portfolios according to each of the four performance measures. The results reveal that all performance measures lead to a similar result. For the full sample period, with some oversimplification, the ranking among the real assets is led by shipping, followed by infrastructure, and with both commodities and real estate at the bottom of the list. But even the worst performing portfolio with a real asset component outperforms the base portfolio.

Exhibit 3 about here

However, this is just the finding for the full sample period, i.e., only for one of our 13 time frames. We conduct the same calculations for the remaining periods. It would simplify the discussion of the results very much, if the ranking according to the four performance measures were highly similar not only for the full sample period, but also for all time frames. To investigate this issue, we look at the correlations among the performance measures for each time frame. We use rank correlation coefficients according to Spearman which are more robust when applied to data which are by the majority not normally distributed. In view of the results in Exhibit 3, the very high correlation coefficients for the full sample period are no surprise (Exhibit 4). And they are no exception. Not all results can be shown to save space, but to demonstrate the generally high level of correlation, the results for 2008 are presented. 2008 is the time frame with the lowest correlation, but it is still remarkably high.

Exhibit 4 about here

Since the results for all four performance measures are so closely correlated with each other, it suffices to exhibit the results for a single performance measure for all time frames. We choose the Sharpe ratio for this purpose. For the sake of clarity, we do not exhibit the absolute value in Exhibit 5, but the portfolio ranking according to the absolute values.

Exhibit 5 about here

How the numbers come about can be traced back by following how the portfolio ranking for the full sample period in Exhibit 3 is transferred to Exhibit 5. The results vary very much for the different time frames. Exhibit 6 provides data to evaluate the extent of intertemporal stability of portfolio rankings.

Exhibit 6 about here

To explain the information given in Exhibit 6, the two figures for the combination of base portfolio and Sharpe ratio can be taken as an example. They state, that over all 13 time frames the base portfolio was ranked at position 12.92 with a standard deviation of 4.23. The smaller the standard deviation, the greater intertemporal stability. The numbers necessary to calculate the figures for the Sharpe ratio can be taken from Exhibit 5.

Discussion

It is advantageous to start the discussion with our second and our third hypothesis because they are concerned with the fundamental question to what extent results found for specific performance measure for a specific time frame can be generalized to other measures and time frames, respectively. As far as the performance measures are concerned, the evidence is clear without ambiguity. The strong correlation among the performance measures shown in Exhibit 3 as well as the apparent similarity of the average ranking for a given portfolio for the performance measures in Exhibit 6 strongly support hypothesis 2: The selection of the performance measure does not matter very much. This statement holds although the distribution characteristics of the return data vary across the sample periods (Exhibit 2) and in theory applicability of the performance measures depends on data distribution.

Thus, our results can be generalized for performance measures, but the opposite is true for time frames. The time frame selected for the analysis matters very much. This can be derived from Exhibit 5 which only displays data for the Sharpe ratio, but the findings are valid for the other measures as well. Additionally, the standard deviations shown in Exhibit 6 also document large differences between the periods analyzed. Thus, hypothesis 3 has to be rejected.

Despite this strong time dependence of results, some more general, though still time sensitive, conclusions can be drawn. With regards to hypothesis 1, Exhibit 6 clearly shows that the base portfolio is positioned at the very bottom of the portfolio ranking according to each performance measure. This is a strong support for the conclusion that the addition of real assets in the majority of the cases improved the mean-risk profile of the portfolio return. Turning to the four real assets, an obvious trend can be identified as well: Infrastructure and shipping make up a group which is clearly superior to the group made up of commodities and real estate. The difference between the groups is much wider than the difference within the groups. In an intra-group perspective, infrastructure slightly outperforms shipping, and commodities are slightly better than real estate. The foundation of our general results for the four real assets is much more solid than the simple result just for the time period from 1999 to 2009 because we integrate 12 sub-periods into our analysis. Still, they reflect past performance which might be unconnected to future performance. Finally, the results for the financial crisis also clearly demonstrate that low correlation — the precondition for an asset's ability to improve the mean-risk profile of a portfolio — breaks down in such a crisis situation.

CONCLUSION

This article takes the perspective of investors who look with great concern at the inflationary potential included in the rescue measures taken by governments and central banks to fight the financial crisis. They often get the advice to invest a larger part of their wealth in real assets because real assets are said to offer at least a partial hedge against inflation. A further consequence of the financial crisis is the increased attractiveness of liquidity. For this reason, we confine the spectrum of real assets on liquid ones, i.e., on exchange-traded investment vehicles. More specifically, we investigate real estate, commodities, infrastructure, and shipping as liquid real assets.

We explore how the addition of these real assets to a portfolio of standard stocks and bonds affects the mean-risk profile of portfolio returns. The assessment of this issue requires to choose from a great variety of performance measures and from an almost innumerous number of time frames for past returns which enter the analysis. One of our main results is that, at least in our context, the selection of the performance measure is irrelevant. Despite possessing great conceptual differences, Sharpe ratio, Sortino ratio, Omega ratio, and Modified Sharpe ratio lead to the same conclusions. This is good news for investors because results are robust with respect to the performance measure chosen.

In contrast to that, the time frame chosen for the analysis matters very much. This is bad news for investors because there is no such thing as the single 'true' time frame for this purpose. Nevertheless, we are able to derive some rather solid conclusions regarding our specific assets, i.e., quite reliable at least for the time period from 1999 to 2009: In most cases, the addition of real assets improved portfolio performance in comparison with a base portfolio consisting only of standard stocks and bonds. Among the four real assets, infrastructure and shipping clearly outperformed commodities and real estate.

REFERENCES

Amenc, N., L. Martellini, and V. Ziemann. "Alternative Investments for Institutional Investors: Risk budgeting techniques in Asset Management and Asset-Liability Management." EDHEC-Risk Asset Management Research, January 2009, pp. 1-76.

Amin, G.S., and H.M. Kat. "Diversification and Yield Enhancement with Hedge Funds." Journal of Alternative Investments, Vol. 5, No. 3 (2002), pp. 50-58.

Amin, G.S., and H.M. Kat. "Stocks, Bonds and Hedge Funds. Not a Free Lunch." Journal of Portfolio Management, Vol. 29, No. 4 (2003), pp. 113-120.

Anson, M.J.P. Handbook of Alternative Assets, 2nd ed. New Jersey: Wiley, 2006.

Bacmann, J.-F., and S. Scholz, S. "Alternative Performance Measures for Hedge Funds." AIMA Journal, 1 (2003), pp. 1-9.

Blum, P., M. Dacorogna, and L. Jaeger. "Performance and Risk Measurement Challenges For Hedge Funds: Empirical Considerations". 2003. Available at http://ideas.repec.org/p/wpa/wuwpri/0311001.html#provider, visited at 23.12.2009.

Bräuninger, M., S. Stiller, and H. Vöpel. “Langfristige Perspektiven von Anlagen in Sachwerten.” HWWI 2009. Available at www.hwwi.org/Publikationen_Einzel.5119.0.html?&tx_wilpubdb_pi1[back]=484&tx_wilpubdb_pi1[publication_id]=1592&cHash=e87137767b, visited at 21.12.2009.

Brounen, D., and P. Eichholtz. “Property, Common Stocks, and Property Shares.” Journal of Portfolio Management, Special Edition (2003), pp. 129-137.

Chan S.H., J. Erickson, and K. Wang. Real Estate Investment Trusts: Structure, Performance, and Investment Opportunities, Oxford and New York: Oxford University Press, 2003.

DeFusco, R.A., D.W. McLeavey, J.E. Pinto and D. E. Runkle. Quantitative Methods For Investment Analysis, 2nd ed. Baltimore: CFA Institute, 2004.

De Souza, C., and S. Gokcan. “Hedge Fund Investing: a Quantitative Approach to Hedge Fund Manager Selection and De-Selection”. Journal of Wealth Management, Vol. 6, No. 4 (2004), pp. 52-73.

Drobetz, W., D. Schilling, and L. Tegtmeier ”Common Risk Factors in the Returns of Shipping Stocks.“ Maritime Policy & Management, Vol. 37, No. 2 (2010), pp. 93-120.

Eagar, D. “Is Inflation the Ultimate Inflation Proof Asset Class?” Portfolio Construction Conference Sydney 2008. Available at http://portfolioconstruction.com.au/obj/articles_pcc08/PCC08_DDF_MAGELLAN.pdf, visited at 11.6.2010.

Emery, K. “Private Equity Risk and Reward: Assessing the Stale Pricing Problem.” Journal of Private Equity, Vol. 6, No. 2 (2003), pp. 43-50.

Favre, L., and J.A. Galeano. “Mean-Modified Value-at-Risk with Hedge Funds.” Journal of Alternative Investments, Vol. 5, No. 2 (2002), pp. 21-25.

French, C.W. “Portfolio Selection with Hedge Funds.” Working Paper, April 2005. Available at ssrn.com, id=757892, visited at 15.5.2010.

Global Property Research (GPR). “GPR 250 Properties Securities Index.” Available at: www.propertyshares.com/algemeen/products/indicesGpr250Index.asp, visited at 21.12.2009.

Gong, S.X.H., M. Firth, and K. Cullinane. “Beta Estimation and Stability in the US-Listed International Transportation Industry.” Review of Pacific Basin Financial Markets and Policies Vol. 9, No. 3 (2006), pp. 463-490.

Gorton, G., and G. Rouwenhorst. “Facts and Fantasies about Commodity Futures.” Financial Analysts Journal, Vol. 62, No. 2 (2006), pp. 47-68.

Grammenos, C.T., and A. Arkoulis. “Macroeconomic Factors and International Stock Returns.” International Journal of Maritime Economics, Vol. 4 (2002), No. 1, pp. 81-99.

Gregoriou, G.N., and J.P. Gueyie. “Risk-Adjusted Performance of Funds of Hedge Funds Using a Modified Sharpe Ratio.” Journal of Wealth Management, Vol. 6, No. 3 (2003), pp. 77-83.

Grelck, M.B., S. Prigge, L. Tegtmeier, and M. Topalov. “Diversification Properties of Investments in Shipping.” Journal of Alternative Investments, Vol. 12, No. 1 (2009), pp. 55-74.

Hamelink, F., M. Hoesli, and B. MacGregor. “Inflation Hedging vs. Inflation Protection in the US and the UK.” Real Estate Finance, Vol. 14, No. 2 (1997), pp. 63–73.

Heidorn, T., D.G. Kaiser, and A. Muschiol. “Portfoliooptimierung mit Hedgefonds unter Berücksichtigung höherer Momente der Verteilung.” Finanz Betrieb 9 (2007), pp. 371-381.

Hübner, R., M.S. Schwaiger, and G. Winkler. “Indirekte Immobilienanlagen im Portfoliomanagement am Beispiel des deutschen Marktes.” Financial Markets and Portfolio Management, Vol. 18, No. 2 (2004), pp. 181-198.

Hudson-Wilson, S., F.J. Fabozzi, and J.N. Gordon. “Why Real Estate?” Journal of Portfolio Management, Special Real Estate Issue 2003, pp. 12-25.

Inderst, G. “Pension Fund Investment in Infrastructure.” OECD Working Paper on Insurance and Private Pensions No. 32, January 2009. Available at http://www.oecd.org/dataoecd/41/9/42052208.pdf, visited at 18.5.2010.

Jefferies & Company. “Thomson Reuters/Jefferies CRB Index.” Available at: www.jefferies.com/cositemgr.pl/html/ProductsServices/SalesTrading/Commodities/ReutersJefferiesCRB/index.shtml, visited at 21.12.2009.

J.P. Morgan. “GBI Global.” Available at: www.jpmorgan.com/pages/jpmorgan/investbk/solutions/research/FIBondIndex, visited at 21.12.2009.

Kavussanos, M.G., A. Juell-Skielse, and M. Forrest. “International Comparison of Market Risks across Shipping-Related Industries.” Maritime Policy & Management, Vol. 30, No. 2 (2003), pp. 175-184.

Liu, C., D. Hartzell, and M. Hoesli. “International Evidence on Real Estate Securities as an Inflation Hedge.” Real Estate Economics, Vol. 25, No. 2 (1997), pp. 193–221.

LPX Group. “NMX 30 Infrastructure Global.” Available at: www.lpx-group.com/nmx/indices/nmx30.html, visited at 21.12.2009.

Mansour, A., and H. Nadji. “Performance Characteristics of Infrastructure Investments”. RREEF Research (2007). Available at www.irei.com/uploads/marketresearch/95/marketResearchFile/PerfCharInfInv.pdf; visited at 11.6.2010.

Maurer, R., and S. Sebastian. “Immobilienfonds und Immobilienaktien als finanzwirtschaftliche Substitute für Immobiliendirektanlagen.” Zeitschrift für Betriebswirtschaft, Ergänzungsheft No. 3 (1999), pp. 169-194.

Milner, F., and E. Vos. “Private Equity: A Portfolio Approach.” Journal of Alternative Investments, Vol. 5, No. 4 (2003), pp. 51-65.

Mull, S., and L. Soennen. ‚S. “U.S. REITs as an Asset Class in International Investment Portfolios.” Financial Analysts Journal, Vol. 53, No. 2 (1997), pp. 55–61.

O´Donnell III, E. “Real Assets and Inflation Hedge Investing.”August (2009), pp. 1-9. Available at www.nepc.com/uploads/research_papers/09_08_real_assets_investing.pdf, visited at 21.12.2009.

Poddig, T., H. Dichtl, and K. Petersmeier. Statistik, Ökonometrie, Optimierung, Bad Soden, Germany: Uhlenbruch, 2000.

Sharpe, W.F. “Mutual Fund Performance.” Journal of Business, Vol. 39, No. 1 (1966), pp. 119-138.

Sortino, F.A., and L.N. Price. ”Performance Measurement in a Downside Risk Framework.” The Journal of Investing, Vol. 3, No. 3 (1994), pp. 59-64.

Stevenson, S. “International Real Estate Diversification: Empirical Tests Using Hedged Indices.” Journal of Real Estate Research, Vol. 19, No. 1 (2000), pp. 105–131.

Wahrenburg, M., D. Schmidt, and R. Reinhard. “Analyse von öffentlich gehandelten Private Equity Instrumenten aus Sicht des Asset Managements.” Working Paper, May 2003. Available at www.cepres.com/Downloads/Publications/CEPRES_Research_Summary_PublicPE.pdf, visited at 15.5.2010.

Exhibit 1: Descriptive Statistics of Monthly Return Data for All Risky Assets for the Full Sample Period from 1999 to 2009

Asset	Observations	Mean	Minimum	Maximum	Standard Deviation	Skewness	Excess Kurtosis	Jarque-Bera
MSCI World	132	0.0030	-0.1893	0.1132	0.0470	-0.77	1.55	26.08***
JPM GBI	132	0.0049	-0.0494	0.0733	0.0216	0.17	0.12	0.73
GPR 250	132	0.0049	-0.2799	0.1967	0.0564	-1.32	5.73	219.15***
Shipping	132	0.0263	-0.2583	0.2318	0.0777	-0.49	1.77	22.66***
NMX 30	132	0.0093	-0.1539	0.1241	0.0440	-0.58	1.27	16.31***
CRB Index	132	0.0040	-0.2232	0.1379	0.0452	-0.95	4.48	130.29***

Notes: The assets are: MSCI World: MSCI World Index (developed market equity performance); JPM GBI: J.P. Morgan Global Bond Index (developed market bond performance); GPR 250: GPR 250 Property Securities Index (real estate); Shipping: Self-developed research index (44 stocks of container, tanker, and bulker companies); NMX 30: NMX30 Infrastructure Global (infrastructure); CRB Index: Thomson Reuters/Jefferies CRB Index (commodities). Discrete monthly returns from January 1999 to December 2009. ***/**/* denotes a significance level better than 0.01/0.05/0.10.

Exhibit 2: Test of Normally Distributed Returns for All Sample Periods

Asset	Full Sample Period	Bear Market	Bull Market	Crisis Short	Crisis Long	2002	2003	2004	2005	2006	2007	2008	2009
MSCI World	R***	ND	ND	R***	R***	R***	R**	R***	ND	R***	R***	R***	R***
JPM GBI	ND	ND	ND	R***	R***	R***	R**	R**	ND	R***	R**	R***	R***
GPR 250	R***	ND	R***	R***	R***	R***	ND	R***	R*	ND	R***	R***	R***
Shipping	R***	ND	ND	R***	R***	R***	R**	R***	R**	R***	R***	R***	R***
NMX 30	R***	ND	ND	R***	R***	R***	R*	R**	R*	R***	R***	R***	R***
CRB Index	R***	ND	ND	R***	R***	ND	ND	ND	R***	R**	R*	R***	R**

Notes: Sample periods: full sample period: 1999-2009; bear market period: 31.3.2000-31.3.2003; bull market period: 31.3.2003-31.10.2007; crisis short: 31.5.2008-31.12.2009; crisis long: 30.11.2007-31.12.2009; 2002-2009: separate analyses of each year. Discrete monthly returns analyzed in full sample period, bear market period, and bull market period, discrete daily returns are used for all other sample periods. The assets are defined in the notes accompanying Exhibit 1. ND: The hypothesis of normally distributed returns cannot be rejected. R: Rejection of the hypothesis of normally distributed returns where ***/**/* denotes a significance level better than 0.01/0.05/0.10.

Exhibit 3: Analysis of Diversification Properties for the Complete Sample Period from 1999 to 2009

Portfolio Weights			Performance Measures							
MSCI World	JPM GBI	Real Asset	Sharpe ratio		Sortino ratio		Omega ratio		Mod. Sharpe ratio	
			Abs. Value	Rank	Abs. Value	Rank	Abs. Value	Rank	Abs. Value	Rank
50%	50%	0%	0.045	17	0.061	17	1.289	17	0.027	17
		GPR 250								
45%	50%	5%	0.048	16	0.066	16	1.304	16	0.029	16
40%	50%	10%	0.052	14	0.070	14	1.317	14	0.031	14
35%	50%	15%	0.055	12	0.074	12	1.327	13	0.033	12
30%	50%	20%	0.057	9	0.077	10	1.337	10	0.034	10
		Shipping								
45%	50%	5%	0.085	5	0.118	5	1.427	6	0.053	5
40%	50%	10%	0.122	3	0.174	3	1.567	3	0.077	3
35%	50%	15%	0.156	2	0.228	2	1.701	2	0.101	2
30%	50%	20%	0.186	1	0.278	1	1.830	1	0.123	1
		NMX 30								
45%	50%	5%	0.057	10	0.078	9	1.335	11	0.035	9
40%	50%	10%	0.070	7	0.096	7	1.383	7	0.043	7
35%	50%	15%	0.082	6	0.114	6	1.430	5	0.051	6
30%	50%	20%	0.094	4	0.133	4	1.477	4	0.059	4
		CRB Index								
45%	50%	5%	0.049	15	0.066	15	1.309	15	0.029	15
40%	50%	10%	0.052	13	0.070	13	1.329	12	0.031	13
35%	50%	15%	0.056	11	0.075	11	1.346	9	0.033	11
30%	50%	20%	0.059	8	0.078	8	1.359	8	0.035	8

Notes: This exhibit displays for 17 portfolios the absolute values of our four performance measures and the ranks derived from these values when applied to the full sample period from 1999 to 2009. The assets are defined in the notes accompanying Exhibit 1. The performance measures are defined as follows: The Sharpe ratio is defined as $S_p = \frac{r_p - r_f}{\sigma_p}$ with S^P: Sharpe ratio of portfolio P, r^P: mean return of portfolio P, σ^P: standard deviation of portfolio P, r^f: risk-free rate of return. Since this exhibit is based on calculations using discrete *monthly* return data, the risk-free rate is approximated by 1-*month*-LIBOR on a US$ basis. (If in following exhibits daily returns are used, the risk-free rate is approximated by 1-day-LIBOR.) The Sortino ratio is defined as $SortinoRatio = \frac{r_p - MAR}{DD_p}$ with r^P: mean return of portfolio P, MAR: minimum acceptable return, DD_d: downside deviation. MAR is defined in our calculations as the risk-free rate of return. DD is calculated as: $DD = \sqrt{\frac{\sum_{i=1,\, r_i<MAR}^{n} (r_i - MAR)^2}{n}}$ with DD: downside deviation, n: number of observations, r_i: rate of return of asset i, MAR: minimum acceptable return. The Omega ratio is defined as: $\Omega(t) = \frac{\sum_{i=1}^{n} \max(0; r_i - T)}{\sum_{i=1}^{n} \max(0; T - r_i)}$ with n: number of observations, r_i: rate of return of asset i, T: threshold return (defined as the average of 0% and the risk-free rate). The Modified Sharpe ratio is defined as: $Modified\ Sharpe\ ratio = \frac{r_p - r_f}{MVaR}$ with r^P: mean return of portfolio P, r^f: risk-free rate of return, MVaR: modified value-at-risk. The modified value-at-risk is defined as:

$$MVaR = -(\mu + (z_\alpha + \frac{(z_\alpha^2 - 1) * \lambda_1}{6} + \frac{(z_\alpha^3 - 3z_\alpha) * \lambda_2}{24} - \frac{(2z_\alpha^3 - 5z_\alpha) * \lambda_1^2}{36}) * \sigma)$$

with λ1: skewness, λ2: excess kurtosis, μ: average return, z_α: the α quantile of the standard normal distribution, σ: standard deviation. α amounts in our calculations to 5%.

Exhibit 4: Rank Correlation Coefficients Among the Four Performance Measures

Panel A: Full Sample Period from 1999 to 2009

Performance Measure	Correlation Coefficients			
	Sharpe ratio	**Modified Sharpe ratio**	**Sortino ratio**	**Omega ratio**
Sharpe ratio	1.000	0.998***	0.998***	0.988***
Modified Sharpe ratio		1.000	1.000***	0.985***
Sortino ratio			1.000	0.985***
Omega				1.000

Panel B: Sample Period: 2008

Performance Measure	Correlation Coefficients			
	Sharpe ratio	**Modified Sharpe ratio**	**Sortino ratio**	**Omega ratio**
Sharpe ratio	1.000	0.929***	0.993***	0.951***
Modified Sharpe ratio		1.000	0.936***	0.931***
Sortino ratio			1.000	0.953***
Omega				1.000

Notes: This exhibit displays the Spearman correlation coefficients between our four performance measures when applied to 17 portfolios for a specific time period. ***/**/* denotes Spearman rank correlation coefficients with a p-value better than 0.01/0.05/0.10 (two-sided). The performance measures are defined in the notes accompanying Exhibit 3.

Exhibit 5: Portfolio Ranking According to the Sharpe Ratio for all 13 Time Frames

Portfolio Weights			Sample Period												
MSCI World	JPM GBI	Real Asset	Full Sample Period	Bear Market	Bull Market	Crisis Long	Crisis Short	2002	2003	2004	2005	2006	2007	2008	2009
50%	50%	0%	17	17	11	9	5	17	17	15	17	13	13	5	12
		GPR 250													
45%	50%	5%	16	15	9	12	7	16	12	12	16	10	14	7	14
40%	50%	10%	14	12	10	15	10	14	10	10	14	6	15	12	15
35%	50%	15%	12	8	12	16	13	10	8	9	11	2	16	14	16
30%	50%	20%	9	5	13	17	15	8	7	6	10	1	17	15	17
		Shipping													
45%	50%	5%	5	11	4	10	6	12	5	8	15	12	5	8	7
40%	50%	10%	3	4	3	11	8	6	3	4	12	9	3	13	4
35%	50%	15%	2	2	2	13	11	3	2	2	9	7	2	16	2
30%	50%	20%	1	1	1	14	12	1	1	1	6	5	1	17	1
		NMX 30													
45%	50%	5%	10	14	8	7	4	15	11	11	13	11	12	4	8
40%	50%	10%	7	10	7	5	3	11	9	7	8	8	10	3	6
35%	50%	15%	6	7	6	3	2	7	6	5	5	4	9	2	5
30%	50%	20%	4	3	5	1	1	4	4	3	4	3	7	1	3
		CRB Index													
45%	50%	5%	15	16	14	8	9	13	15	13	7	14	11	6	10
40%	50%	10%	13	13	15	6	14	9	13	14	3	15	8	9	9
35%	50%	15%	11	9	16	4	16	5	14	16	2	16	6	10	11
30%	50%	20%	8	6	17	2	17	2	16	17	1	17	4	11	13

Notes: This exhibit displays the ranking of our 17 portfolios according to the Sharpe ratio. It is a condensed presentation of calculations as in Exhibit 3. The assets are defined in the notes accompanying Exhibit 1. The sample periods are defined in the notes accompanying Exhibit 2. The Sharpe ratio is defined in the notes accompanying Exhibit 3.

Exhibit 6: Mean and Standard Deviation of the Four Performance Measures over the 13 Time Frames

Portfolio Weights			Performance Measures							
MSCI World	JPM GBI	Real Asset	Sharpe ratio		Sortino ratio		Omega ratio		Mod. Sharpe ratio	
			Mean	Stand. Dev.	Mean	Stand. Dev.	Mean	Stand. Dev.	Mean	Stand. Dev.
50%	50%	0%	12.92	4.23	12.77	4.34	12.85	4.38	12.54	4.97
		GPR 250								
45%	50%	5%	12.31	3.12	12.38	3.05	12.54	3.13	12.38	3.05
40%	50%	10%	12.08	2.64	12.15	2.77	12.00	2.66	12.15	2.77
35%	50%	15%	11.31	3.85	11.46	3.91	11.46	3.59	11.38	3.87
30%	50%	20%	10.77	5.07	11.15	5.13	10.85	5.11	11.08	5.06
		Shipping								
45%	50%	5%	8.31	3.29	8.38	3.25	8.54	3.25	8.46	3.41
40%	50%	10%	6.38	3.61	6.31	3.67	6.69	3.69	6.31	3.67
35%	50%	15%	5.62	4.83	5.54	4.67	5.69	4.84	5.62	4.83
30%	50%	20%	4.77	5.56	4.85	5.57	4.92	5.59	4.85	5.57
		NMX 30								
45%	50%	5%	9.85	3.35	9.77	3.35	10.00	3.44	9.62	3.65
40%	50%	10%	7.23	2.42	7.31	2.40	7.15	2.41	7.31	2.40
35%	50%	15%	5.15	1.96	5.15	1.96	5.00	1.88	5.31	1.77
30%	50%	20%	3.31	1.64	3.38	1.64	3.23	1.67	3.77	1.58
		CRB Index								
45%	50%	5%	11.62	3.20	11.38	3.13	11.77	3.14	11.54	3.10
40%	50%	10%	10.85	3.63	10.62	3.54	11.00	3.64	10.85	3.53
35%	50%	15%	10.46	4.81	10.46	4.91	9.92	4.68	10.23	4.92
30%	50%	20%	10.08	6.24	9.92	6.29	9.38	6.31	9.62	6.26

Notes: This exhibit displays mean and standard deviation of our four performance measures over 13 time frames for a specific portfolio. The assets are defined in the notes accompanying Exhibit 1. The sample periods are defined in the notes accompanying Exhibit 2. The performance measures are defined in the notes accompanying Exhibit 3.